Time-Dependent Reliability Analysis of a Slider-Crank Mechanism System

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1. Slider-crank mechanism system

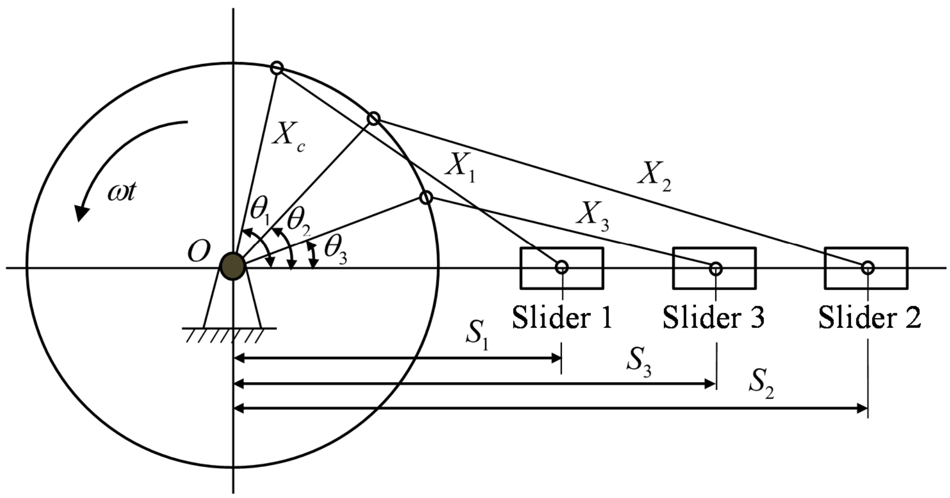

Fig. 1 A system of slider-crank mechanisms

Fig. 1 shows a system with three slider-crank mechanisms. The three cranks are attached to the disc by revolute joints, and the three cranks therefore have the same angular velocity and the same length, which is the radius of the disc X_c. The angular velocity is $\omega = 1$ rad/s. The lengths of the three couplers are X_1, X_2, and X_3, respectively.

The motion outputs are the displacements of the three sliders, denoted by S_i ($i = 1,2,3$). They are given by

$$S_i = X_c \cos\theta_i + \sqrt{X_i^2 - (X_c \sin\theta_i)^2} \quad (1)$$

where θ_i are the motion inputs as shown in Fig. 1. The required motion outputs are the nominal displacements of the sliders and are given by

$$S_{R_i} = \mu_c \cos\theta_i + \sqrt{\mu_i^2 - (\mu_c \sin\theta_i)^2} \quad (2)$$

where μ_c and μ_i are the mean values of X_c and X_i, respectively.

The motion errors are defined by

$$\Delta S_i = \left| S_{R_i} - S_i \right| \quad (3)$$

2. Reliability definition and information about random variables

Time-dependent system reliability analysis evaluates the probability of failure $p_f^S(t_0, t_s)$ of a system after it has been put into operation for a period of time defined by $[t_0, t_s]$. For the slider-crank mechanism system problem, the system is required to produce accurate motion output within a full motion cycle, and the period of time is therefore $[0, 2\pi]$ seconds. There are three limit-state functions given by

$$\Delta S_i = Y_i = g_i(\mathbf{X}, t) \tag{4}$$

in which $\mathbf{X}$ is a vector of random variables, and t is time. For this problem, $\mathbf{X} = (\mathrm{X}_c, \mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3)$.

The probability of failure of mechanism i is given by

$$\mathrm{p}_f^i(t_0, t_s) = \Pr\{g_i(\mathbf{X}, t) > 0,\ \exists\, t \in [0, 2\pi]\} \tag{5}$$

where $\Pr\{\cdot\}$ stands for a probability, and $\exists$ stands for "there exists".

For the present series system, $p_f^S(t_0, t_s)$ is defined by

$$p_f^S(t_0, t_s) = \Pr\{g_1(\mathbf{X}, t) > 0 \cup g_2(\mathbf{X}, t) > 0 \cup g_3(\mathbf{X}, t) > 0,\ \exists\, t \in [0, 2\pi]\} \tag{6}$$

For this mechanism system, all the lengths are independent random variables, and one possible set of their distributions is given in Table 1.

Table 1 Random variables

Variable	Mean (mm)	Standard deviation (mm)	Distribution
X_c	100	0.5	Normal
X_1	150	0.75	Normal
X_2	250	1.25	Normal
X_3	200	1.0	Normal

The motion errors of the mechanisms should not be greater than the allowable motion errors ε_i. One example of the possible allowable motion errors is as follows: $\varepsilon_1 = 4.8$ mm, $\varepsilon_2 = 5.5$ mm, and $\varepsilon_3 = 5.2$ mm, respectively. Given the motion inputs to be $\theta_1 = \omega t$, $\theta_2 = \omega t - \pi/6$, and $\theta_3 = \omega t - \pi/3$, the limit-state functions are

$$Y_i = g_i(\mathbf{X}, t) = \left|(X_c - \mu_c)\cos\theta_i + \sqrt{X_i^2 - (X_c \sin\theta_i)^2} - \sqrt{\mu_i^2 - (\mu_c \sin\theta_i)^2}\right| - \varepsilon_i \tag{7}$$

3. Features of the limit-state functions

The limit-state functions are quite nonlinear with respect to time. One trajectory of the motion error of the first mechanism is plotted in Fig. 2 for $(X_c, X_1) = (100.5, 150.0)$ mm. As shown in Fig. 2, the derivatives of the limit-state function at two time instants do not exist. This makes it difficult to use traditional reliability methods, for example, the First and Second Order Reliability Methods (FORM and SORM). The maximum values of limit-state functions with respect to time are required to calculate the time-dependent probabilities of failure. Fig. 3 shows the maximum motion error of mechanism 1. The shape of the maximum error is quite irregular. Fig. 4 shows the probability density function of the maximum error of mechanism 1.

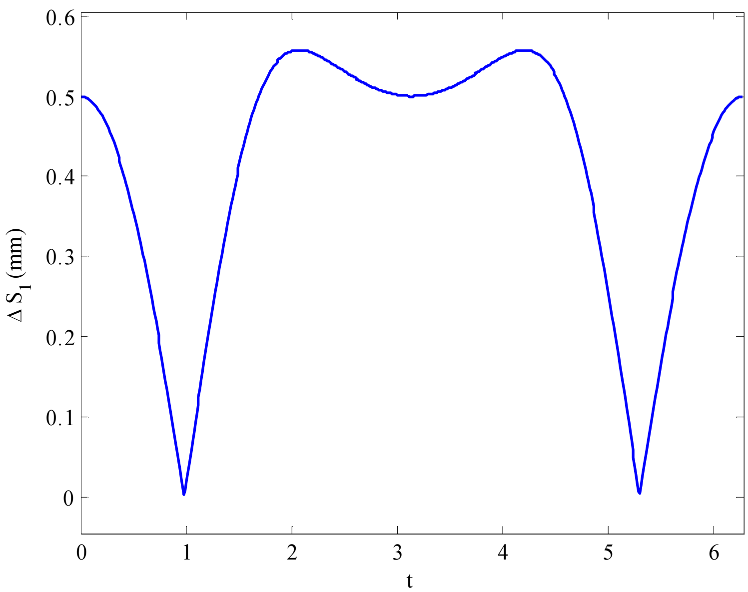

Fig. 2 Motion error of mechanism 1 at point (100.5, 150.0) mm

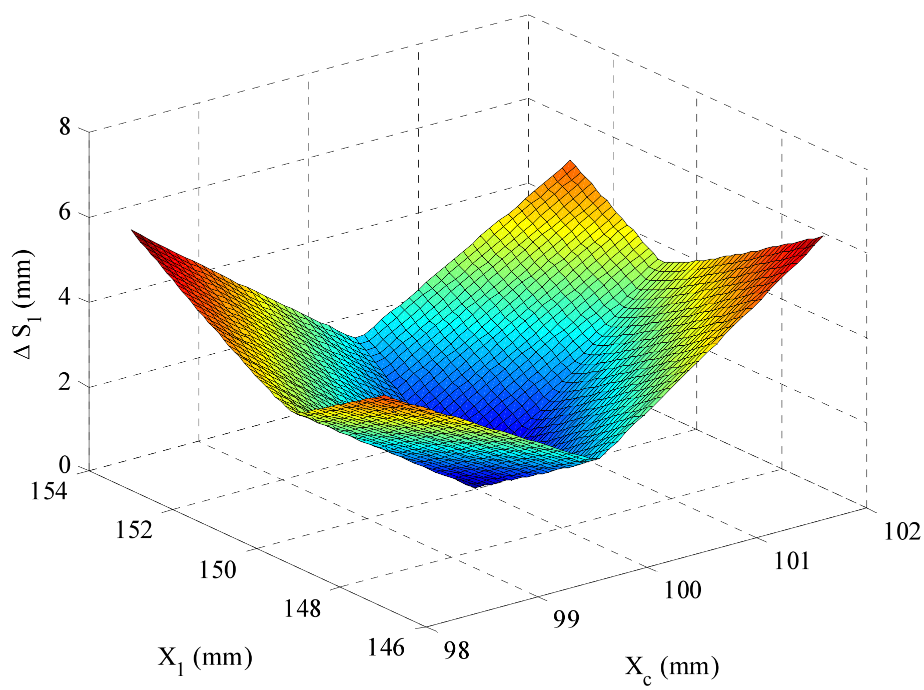

Fig. 3 Maximum motion error of mechanism 1

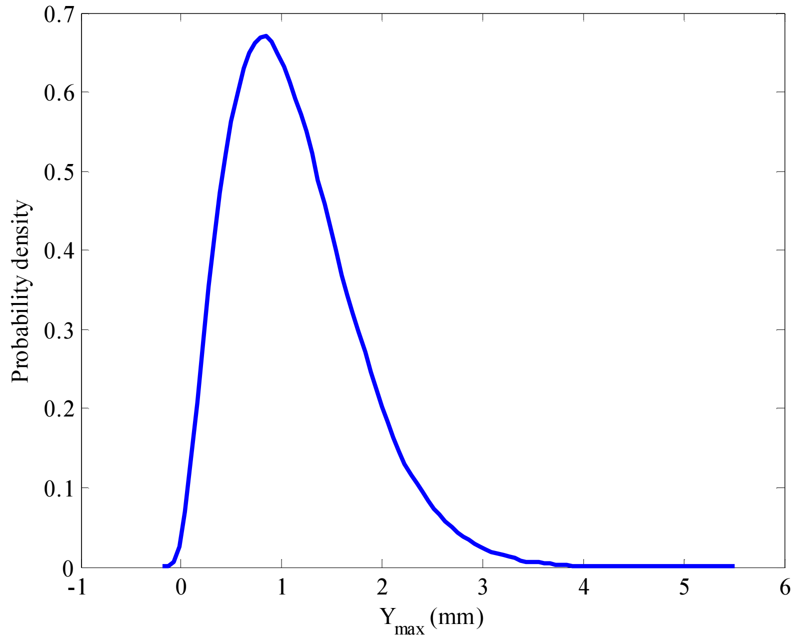

Fig. 4 Probability density function of the maximum motion error of mechanism 1

4. Matlab codes

   The Matlab codes of this problem are downloadable at:

   http://web.mst.edu/~dux/repository/computer_codes.html.

The function and its inputs and outputs are described below.

(1) Limit-state functions: limit_state_functions.m

```
[Y1,Y2,Y3]=limit_state_functions(t,X,X_mu,omega,epsilon)
```

Inputs:

- t: a row vector of time instants.
- X: a row vector of lengths of links (X_c, X_1, X_2, X_3).
- X_mu: a row vector of mean values of X.
- omega: the angular velocity of the three cranks.
- epsilon: a row vector of allowable motion errors.

Outputs:

- Y1: a row vector of limit-state function values of mechanism 1.
- Y2: a row vector of limit-state function values of mechanism 2.
- Y3: a row vector of limit-state function values of mechanism 3.

(2) Example

If one wants to obtain the limit-state functions at 0, 1 and 5 seconds, the lengths are $X_c = 100.1$ mm, $X_1 = 149.8$ mm, $X_2 = 250.5$ mm and $X_3 = 199.6$ mm; $\omega = 1$ rad/s; and $\varepsilon_1 = 0.20$ mm, $\varepsilon_2 = 0.40$ mm and $\varepsilon_3 = 0.35$ mm, then the code is as follows:

```
t=[0, 1, 5];
X=[100.1, 149.8, 250.5, 199.6];
X_mu=[100.0, 150.0, 250.0, 200.0];
omega=1;
epsilon=[0.20, 0.40, 0.35];
[Y1,Y2,Y3]=limit_state_functions(t,X,X_mu,omega,epsilon)
```

Then the outputs will be

```
Y1=[-0.1000, 0.0448, 0.1118].
Y2=[0.1867, 0.1889, 0.0783].
Y3=[0.0856, -0.0497, 0.1764].
```